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Free vibration analysis of segmented Timoshenko beams on Pasternak foundation by using transfer matrix

Transfer matrisi kullanılarak Pasternak zemini üzerine oturan kademeli Timoshenko kirişlerinin serbest titreşim analizi

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Abstract

The aim of this study is performing free vibration analysis of segmented Timoshenko beams on two parameter elastic foundation by using the transfer matrix method (TMM). The Pasternak foundation model which has an incompressible shear layer of vertical elements attached to the Winkler springs was considered. The transfer matrix formulations which are based on closed-form solutions of equations of motion of Timoshenko beams on Pasternak foundation were obtained. The natural frequencies were calculated by equating the determinant of global transfer matrix of structure to zero after the reduction according to boundary conditions. The mode shapes were plotted by normalising the state vectors at the ends. Firstly, the natural frequencies that obtained by using the proposed approach were validated by data in literature for a simply supported beam where a very good agreement was observed. Then, three-segmented beam models having various boundary conditions namely simple-simple (S-S), simple-fixed (S-F), fixed-simple (F-S) and fixed-fixed (F-F) were considered for numerical analysis. For the segmented beam models, the natural frequencies that calculated by using the TMM were compared to the results of finite element method (FEM) from SAP2000 by ignoring effects of shear layer of elastic foundation. The effects of shear layer as well as stiffness of Winkler springs on the natural frequencies of the segmented beam model were revealed for S-S, S-F, F-S and F-F boundary conditions, respectively. The mode shapes of the segmented beam model were presented. The results show that TMM can be used as an effective tool for free vibration analysis of multi-segmented Timoshenko beams on Pasternak foundation.

Keywords: *Mode shape, Natural frequency, Pasternak foundation, Segmented Timoshenko beam, Transfer matrix*

Öz

Bu çalışmada, iki parametreli elastik zemin üzerine oturan kademeli Timoshenko kirişlerinin serbest titreşim analizinin transfer matrisi yöntemi (TMM) kullanılarak gerçekleştirilmesi amaçlanmıştır. Winkler yaylarına eklenmiş, sıkıştırılamaz dikey elemanlardan oluşan bir kayma tabakasına sahip olan Pasternak zemin modeli dikkate alınmıştır. Pasternak zemini üzerine oturan Timoshenko kirişlerinin hareket denklemlerinin kapalı çözümlerine dayanan transfer matrisi formülasyonları elde edilmiştir. Doğal frekanslar, sınır şartlarına göre indirgeme yapılması sonrası yapının global

transfer matrisinin determinantının sıfıra eşitlenmesiyle hesaplanmıştır. Mod şekilleri, uçlardaki durum vektörleri normalize edilerek çizilmiştir. İlk olarak, her iki ucu basit mesnetli bir kiriş için, önerilen yaklaşım kullanılarak elde edilen doğal frekanslar literatürdeki verilerle doğrulanmıştır ve çok iyi bir uyum gözlemlenmiştir. Daha sonra, sayısal analiz için basit-basit (S-S), basit-ankastre (S-F), ankastre-basit (F-S) ve ankastre-ankastre (F-F) sınır koşullarına sahip üç kademeli kiriş modelleri dikkate alınmıştır. Kademeli kiriş modeli için TMM kullanılarak hesaplanan doğal frekanslar, elastik zeminin kayma tabakasının etkileri ihmal edilerek SAP2000'in sonlu elemanlar yöntemi (FEM) sonuçlarıyla karşılaştırılmıştır. Winkler yaylarının rijitliğinin ve kayma tabakasının kademeli kiriş modelinin doğal frekansları üzerindeki etkileri sırasıyla S-S, S-F, F-S ve F-F sınır koşulları için ortaya çıkarılmıştır. Kademeli kiriş modelinin mod şekilleri sunulmuştur. Sonuçlar, TMM'nin Pasternak zemini üzerine oturan çok kademeli Timoshenko kirişlerinin serbest titreşim analizi için etkili bir araç olarak kullanılabileceğini göstermektedir.
Anahtar Kelimeler: *Mod şekli, Doğal frekans, Pasternak zemini, Kademeli Timoshenko kirişi, Transfer matrisi*

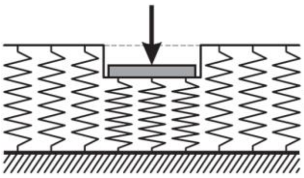
1. Introduction

Beam-assemblies resting on elastic foundation are common structures in the applications of geotechnical, railway and aerospace engineering. Therefore, vibration problem of this type of structures has been an attractive research area in the field. The well known Winkler foundation model which is also named as one-parameter foundation model, is based on representing the foundation as unconnected closely spaced linear elastic springs. This approach is very common and simple but cannot represent the continuity characteristics of foundation. One of the important alternative foundation model to overcome this issue is Pasternak model which considers an incompressible shear layer attached to the Winkler springs. The continuity of elastic foundation is represented by the shear layer in the Pasternak foundation model [1]. The behaviour of Winkler and Pasternak elastic foundations can be seen from Figure 1 comparatively.

The free vibrations of various types of beam/beam-assemblies on Winkler foundation were extensively investigated by many researchers [2-8]. However, the studies on the free vibration analysis of Timoshenko beams on Pasternak foundation are limited when compared to research considering Winkler foundation. Yokoyama [9] analytically calculated the first five natural frequencies of simply supported Timoshenko beams on Pasternak foundation. Wang et al. [10] obtained fundamental frequency parameters of single-span Timoshenko beams on Pasternak foundation having various boundary conditions by using Green's functions. Caliò and Greco [11] investigated free vibrations of Timoshenko beam-columns on Pasternak foundation by using the dynamic stiffness method (DSM). Tonzani and Elishakoff [12] presented three alternative versions of Timoshenko beams on Pasternak foundation for free vibration analysis considering various boundary conditions.

The analytical based transfer matrix formulations are used effectively for investigating the free vibrations of beam-assembly structures. The TMM uses the transfer matrices based on the relation between the state vectors of each end of beams. After the construction of transfer matrix of each segment, the overall transfer matrix of the whole vibrating system is obtained by a chain-multiplication. Then, free vibration analysis of the system can be performed. The TMM has been effectively used for the free vibration analysis of various type of structures such as tapered beams [13], rotating beams [14], multi-span beams with coupled rigid bodies [15], cracked beams [16, 17] and single-span cracked frames [18].

In this study, the TMM was applied to free vibration analysis of multi-segmented Timoshenko beams resting on Pasternak foundation. Firstly, the natural frequency values that obtained by using TMM for a simply supported Timoshenko beam on Pasternak foundation were presented in comparison with data in literature for verification. Then, a three-segmented Timoshenko beam on Pasternak foundation was considered for numerical case study. The particular case of Winkler foundation which can be obtained by ignoring shear layer of Pasternak foundation, was considered to present the results of TMM with the results of SAP2000 for comparison purposes. After the validation of the TMM approach for calculating the natural frequencies of multi-segmented Timoshenko beams on one-parameter foundation, the free vibration analysis was performed for the three-segmented Timoshenko beam on Pasternak foundation considering S-S, S-F, F-S and F-F boundary conditions. The calculated natural frequencies were presented in tables to observe the effects of shear layer as well as spring stiffness of the elastic foundation. Finally, the mode shapes were presented for various boundary conditions.



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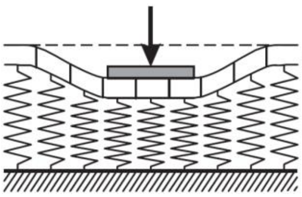
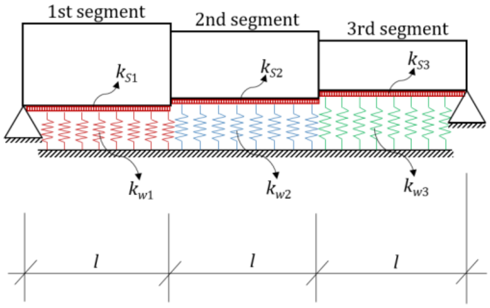


(b)

Figure 1. Displacement of a) Winkler b) Pasternak foundation models [1]

2. Model and formulation

The mathematical models of segmented Timoshenko beams on Pasternak foundation are presented in Figures 2a-d for S-S, S-F, F-S and F-F boundary conditions, respectively. In Figures 2a-d, l is length of each beam segment, k_{S1}, k_{S2}, k_{S3} and k_{w1}, k_{w2}, k_{w3} represent modulus of shear layer and modulus of Winkler foundation, respectively. The following assumptions are considered: i) The material of the beams is homogenous and isotropic ii) the behaviour of the beams is linear and elastic iii) the effects of damping are neglected.



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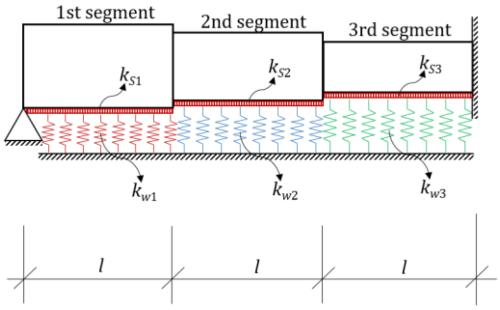

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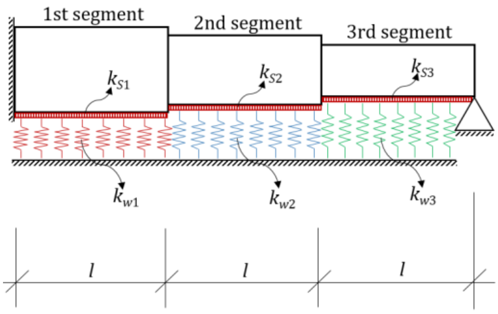

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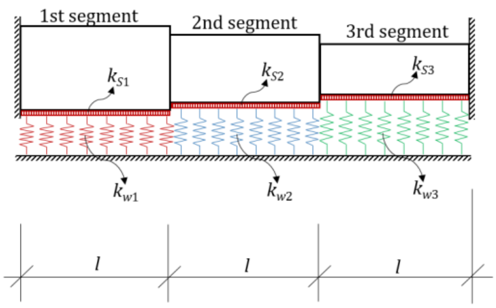

(d)

Figure 2. Mathematical model of segmented Timoshenko beam on Pasternak foundation for a) S-S b) S-F c) F-S d) F-F boundary conditions

The equations of motion of a Timoshenko beam on Pasternak foundation are presented in Eqs.(1)-(2) [11, 12].

$$AG\bar{k}\left(\frac{\partial^2 Y(x,t)}{\partial x^2}-\frac{\partial\theta(x,t)}{\partial x}\right)-\rho A\frac{\partial^2 Y(x,t)}{\partial t^2} -k_w Y(x,t)+k_S\frac{\partial^2 Y(x,t)}{\partial x^2}=0 \quad (1)$$

$$EI\frac{\partial^2\theta(x,t)}{\partial^2 x}+AG\bar{k}\left(\frac{\partial Y(x,t)}{\partial x}-\theta(x,t)\right) -\rho I\frac{\partial^2\theta(x,t)}{\partial t^2}=0 \quad (2)$$

where A is cross-sectional area, E is modulus of elasticity, G is shear modulus, ρ is density, I is area moment of inertia, $\bar{k}$ is shear correction factor, x is beam coordinate, t is time, k_w is modulus of Winkler foundation, k_S is modulus of shear layer, Y(x,t) and θ(x,t) are functions of transverse displacement and bending rotation, respectively.

Assuming the harmonic motion with the assumption of $Y(x,t) = y(x)e^{i\omega t}$ and $\theta(x,t) = \Phi(x)e^{i\omega t}$, Eqs.(3)-(4) are obtained:

$$\frac{AG\bar{k}+k_S}{L^2}\frac{d^2y(z)}{\partial z^2}-\frac{AG\bar{k}}{L}\frac{d\Phi(z)}{\partial z} +\left(\rho A\omega^2-k_w\right)y(z)=0 \quad (3)$$

$$\frac{EI}{L^2}\frac{d^2\Phi(z)}{dz^2}+\frac{AG\bar{k}}{L}\frac{dy(z)}{dz} +\left(\rho I\omega^2-AG\bar{k}\right)\Phi(z)=0 \quad (4)$$

where ω is natural frequency, L is length of the beam and z = x / L.

The solutions of functions y(z) and Φ(z) are assumed as Eq.(5)-(6), respectively.

$$y(z)=\{C_n\}e^{is_n z} \quad (5)$$

$$\Phi(z)=\{D_n\}e^{is_n z} \quad (6)$$

where s_n and n represent the characteristic roots and order of the equation system in Eqs.(3)-(4), $\{C_n\}$ and $\{D_n\}$ are integration constants for the y(z) and Φ(z), respectively.

Eqs.(7)-(8) are obtained by substituting Eqs.(5)-(6) into Eqs.(3)-(4).

$$\left(\rho A\omega^2-k_w-\frac{\left(AG\bar{k}+k_S\right)s_n^2}{L^2}\right)\{C_n\} -\frac{AG\bar{k}is_n}{L}\{D_n\}=0 \quad (7)$$

$$\frac{AG\bar{k}is_n}{L}\{C_n\}+\left(\rho I\omega^2-AG\bar{k}-\frac{EIs_n^2}{L^2}\right)\{D_n\}=0 \quad (8)$$

The characteristic roots s_n are obtained by considering the non-trivial solution of the equation that represents the matrix form of Eqs.(7)-(8).

The functions of y(z) and Φ(z) can be written as follows:

$$y(z)=\sum_{n=1}^{4}\{C_n\}e^{is_n z} \quad (9)$$

$$\Phi(z)=\sum_{n=1}^{4}\{C_n\}K_n e^{is_n z} \quad (10)$$

where

$$K_n=\left(-AG\bar{k}is_n/L\right)/\left(\rho I\omega^2-AG\bar{k}-\left(EIs_n^2/L^2\right)\right),$$
$n=1,2,3,4$

The shear force Q(z) and bending moment M(z) functions are defined in Eqs.(11)-12), respectively [19].

$$Q(z)=\left(AG\bar{k}/L\right)\frac{dy(z)}{dz}-AG\bar{k}\Phi(z) \quad (11)$$

$$M(z)=\left(EI/L\right)\frac{d\Phi}{dz} \quad (12)$$

Substituting Eqs. (9)-(10) into Eqs.(11)-(12), Eqs.(13)-(14) are obtained.

$$Q(z)=\sum_{n=1}^{4}\left(\left(AG\bar{k}is_n/L\right)-\left(AG\bar{k}K_n\right)\right)\{C_n\}e^{is_n z} \quad (13)$$

$$M(z)=\sum_{n=1}^{4}\left(is_n K_n EI/L\right)\{C_n\}e^{is_n z} \quad (14)$$

The transfer matrix formulations can be constructed by using Eqs.(9)-(10) and Eqs.(13)-(14) for each beam segment.

3. Application of TMM

The relation between the state vectors of each end of the beam segments are used to obtain the transfer matrices. The state vector of left-hand side {Z_0} for a beam segment is presented in Eq.(15).

$$\left\{Z_0\right\} = T_0\left\{C\right\} \tag{15}$$

where

$\{Z_0\} = \{y_{z=0}\ \Phi_{z=0}\ Q_{z=0}\ M_{z=0}\}^T$,

$\{C\} = \{C_1\ C_2\ C_3\ C_4\}^T$,

$$T_0 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ K_1 & K_2 & K_3 & K_4 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{bmatrix}$$

$\alpha_n = \left(AG\bar{k}is_n / L\right) - \left(AG\bar{k}K_n\right), \beta_n = \left(is_n K_n EI / L\right),$
$n = 1,2,3,4$

The state vector of right-hand side {Z_1} for a beam segment is defined as follows:

$$\left\{Z_1\right\} = T_1\left\{C\right\} \tag{16}$$

where

$\{Z_0\} = \{y_{z=1}\ \Phi_{z=1}\ Q_{z=1}\ M_{z=1}\}^T$,

$$T_0 = \begin{bmatrix} e^{is_1} & e^{is_2} & e^{is_3} & e^{is_4} \\ K_1 e^{is_1} & K_2 e^{is_2} & K_3 e^{is_3} & K_4 e^{is_4} \\ \alpha_1 e^{is_1} & \alpha_2 e^{is_2} & \alpha_3 e^{is_3} & \alpha_4 e^{is_4} \\ \beta_1 e^{is_1} & \beta_2 e^{is_2} & \beta_3 e^{is_3} & \beta_4 e^{is_4} \end{bmatrix}$$

The relationship between the {Z_0} and {Z_1} are obtained by using Eqs.(15)-(16) as follows:

$$\left\{\mathrm{Z}_1\right\} = TM\left\{\mathrm{Z}_0\right\} \tag{17}$$

where TM is transfer matrix of the beam segment and $TM = T_1 T_0^{-1}$

For the beam models presented in Figures 2a-d, the global transfer matrix of the whole vibrating system is constructed by a chain multiplication of transfer matrices of beam segments as:

$$GTM = TM_3 TM_2 TM_1 \tag{18}$$

where GTM is global transfer matrix of the beam model, TM_3, TM_2 and TM_1 are transfer matrices of third, second and first segment, respectively.

The natural frequencies of Timoshenko beam on Pasternak foundation models are obtained by equating the determinant of reduced global transfer matrix to zero. A bisection based root finding algorithm which applies a trial and error on interpolation can be used for calculation of roots. If there is change of sign between two intervals, there must be root. After some iterations, the natural frequency values are obtained with desired accuracy.

The reduced global transfer matrices which are obtained according to boundary conditions, are presented for S-S, S-F, F-S and F-F in Table 1. The procedure for plotting mode shapes is summarized as follows: i) Using the relation between reduced global transfer matrices and non-zero state vector elements, a normalization is applied ii) substituting zeros according to boundary conditions and normalized values to state vectors given in Eqs.(15)-(16), the mode shapes are obtained iii) the procedure is repeated for each mode.

Table 1. The reduced global transfer matrices for S-S, S-F, F-S and F-F boundary conditions

Boundary condition	Reduced global transfer matrix
S-S	$\begin{bmatrix} GTM(1,2) & GTM(1,3) \\ GTM(4,2) & GTM(4,3) \end{bmatrix}$
S-F	$\begin{bmatrix} GTM(1,2) & GTM(1,3) \\ GTM(2,2) & GTM(2,3) \end{bmatrix}$
F-S	$\begin{bmatrix} GTM(1,3) & GTM(1,4) \\ GTM(4,3) & GTM(4,4) \end{bmatrix}$
F-F	$\begin{bmatrix} GTM(1,3) & GTM(1,4) \\ GTM(2,3) & GTM(2,4) \end{bmatrix}$

4. Verification using the data in literature

The proposed approach for free vibration analysis of Timoshenko beams on Pasternak foundation is validated using the available data in literature for simply supported model. The following non-dimensional parameters were used [1]:

- B/L = 0.2
- h/L = 0.1

- G/E = 0.43
- $K_W = BL^4k_w / EI$
- $K_S = BL^2k_S / EI$

where B represents the cross-sectional width of the beam.

In Table 2, the first three natural frequencies of the simply-supported beam model obtained by using TMM and Runge-Kutta method [1] are presented comparatively. According to Table 2, the agreement between the proposed approach and Runge-Kutta method for calculating natural frequencies of Timoshenko beams on Pasternak foundation is very well for various foundation parameter values. Table 1 shows that first, second and third natural frequencies of simply-supported beam model are increased by 7.9%, 9.8% and 6.1%, respectively, by increasing K_S from 0 to 10.

Table 2. First three natural frequencies (rad/s) of simply-supported Timoshenko beam on Pasternak foundation (K_W = 500)

K_S	Method	1st mode	2nd mode	3rd mode
0	TMM	8.789	15.663	29.593
	[1]	8.791	15.667	29.600
2	TMM	8.934	15.980	29.964
	[1]	8.936	15.984	29.971
4	TMM	9.076	16.292	30.330
	[1]	9.078	16.296	30.337
6	TMM	9.216	16.597	30.692
	[1]	9.218	16.602	30.699
8	TMM	9.354	16.897	31.050
	[1]	9.356	16.901	31.058
10	TMM	9.485	17.192	31.403
	[1]	9.492	17.258	31.411

5. Numerical case study

Numerical analysis was performed for beam models presented in Figures 2a-d. The material and geometrical properties of the segmented beam models are presented in Table 3.

Table 3. Material and geometrical properties of three-segmented Timoshenko beam on Pasternak foundation models

Segment	1st	2nd	3rd
E (kN/m²)	3×10^7	3×10^7	3×10^7
G (kN/m²)	1.25×10^7	1.25×10^7	1.25×10^7
B (m)	1	1	1
H (m)	0.5	0.4	0.3
A (m²)	0.5	0.4	0.3
I (m⁴)	0.0104	0.0053	0.0022
ρ (kg/m³)	2500	2500	2500
l (m)	2.5	2.5	2.5
$\bar{k}$	0.833	0.833	0.833

For the validation of TMM on free vibration analysis of segmented Timoshenko beams on elastic foundation, several finite element analyses were performed by using SAP2000 where k_S is neglected due to limitations of software. It should be noted that the mesh option was set to 50 elements for each segment in SAP2000.

The first three natural frequencies of three-segmented Timoshenko beams on Winkler foundation model are presented in Table 4 for TMM and FEM comparatively. According to Table 4, there is a very good agreement between the results of TMM and FEM for S-S, S-F, F-S and F-F boundary conditions. Table 4 shows that ignoring elastic foundation results in unacceptable errors for the natural frequencies of segmented Timoshenko beams on Winkler foundation by taking K_W = 100.

Table 4. First three natural frequencies (rad/s) of segmented Timoshenko beams on Winkler foundation

Model	K_w	Method	1st mode	2nd mode	3rd mode
S-S	0	TMM	64.545	260.811	598.916
		FEM	64.613	261.937	603.892
	100	TMM	560.185	710.630	896.088
		FEM	561.472	713.137	902.777
S-F	0	TMM	97.217	327.657	686.425
		FEM	97.345	329.130	693.019
	100	TMM	609.129	737.536	952.988
		FEM	610.173	740.760	961.200
F-S	0	TMM	109.975	328.260	689.403
		FEM	110.095	329.782	694.604
	100	TMM	560.228	717.731	949.421
		FEM	561.509	720.555	956.540
F-F	0	TMM	151.897	397.239	787.934
		FEM	152.081	399.902	794.431
	100	TMM	609.131	750.460	1025.415
		FEM	610.177	754.154	1033.304

It should be noted that Pasternak foundation parameters for segmented beam models are calculated by using the nondimensional K_W and K_S values as follows:

$$k_{w1}=(EIK_W)/(B_1l^4), k_{w2}=(EIK_W)/(B_2l^4),$$
$$k_{w3}=(EIK_W)/(B_3l^4), k_{S1}=(EIK_S)/(B_1l^2), \quad (19)$$
$$k_{S2}=(EIK_S)/(B_2l^2), k_{S3}=(EIK_S)/(B_3l^2)$$

where B_1, B_2 and B_3 are cross-sectional width of first, second and third segment of the beam and can be found in Table 3.

The first three natural frequencies of segmented Timoshenko beams on Pasternak foundation that obtained by using the TMM were presented in Tables 5-8 for S-S, F-S, S-F and F-F boundary conditions, respectively.

Table 5. First three natural frequencies (rad/s) of segmented Timoshenko beams on Pasternak foundation for S-S boundary condition

K_w	K_s	1st mode	2nd mode	3rd mode
50	2	426.997	558.769	806.193
	4	437.301	583.244	852.344
	6	447.073	607.211	895.943
75	2	505.940	649.563	873.002
	4	516.147	669.680	915.204
	6	525.707	689.733	955.477
100	2	571.136	728.168	936.316
	4	581.329	745.748	975.012
	6	590.869	763.332	1012.311

Table 6. First three natural frequencies (rad/s) of segmented Timoshenko beams on Pasternak foundation for F-S boundary condition

K_w	K_s	1st mode	2nd mode	3rd mode
50	2	427.361	581.670	873.208
	4	438.077	606.716	916.079
	6	448.299	631.007	956.988
75	2	505.958	663.928	932.207
	4	516.278	685.436	972.075
	6	526.024	706.549	1010.402
100	2	571.135	737.141	987.983
	4	581.336	756.285	1025.257
	6	590.937	775.171	1061.326

Table 7. First three natural frequencies (rad/s) of segmented Timoshenko beams on Pasternak foundation for S-F boundary condition

K_w	K_s	1st mode	2nd mode	3rd mode
50	2	463.046	594.888	874.608
	4	471.788	621.445	920.915
	6	480.406	646.915	964.679
75	2	547.884	680.762	936.098
	4	555.933	702.921	979.365
	6	563.745	724.615	1020.487
100	2	617.336	756.877	995.439
	4	625.196	776.088	1035.845
	6	632.752	795.110	1074.491

Table 8. First three natural frequencies (rad/s) of segmented Timoshenko beams on Pasternak foundation for F-F boundary condition

K_w	K_s	1st mode	2nd mode	3rd mode
50	2	465.066	625.430	956.771
	4	474.454	651.381	999.160
	6	483.640	676.309	1039.606
75	2	548.354	701.959	1011.847
	4	556.771	724.708	1051.818
	6	564.968	746.801	1090.145
100	2	617.412	771.351	1064.733
	4	625.428	791.751	1102.509
	6	633.190	811.701	1138.903

It can be seen from Tables 5-8 that the increasing K_S values slightly increases the first three natural frequencies for constant K_W values. However, a significant augmentation is observed by an increment for the K_W from 50 to 75 and 100 respectively, for each boundary condition considered in the numerical case study. According to Tables 5-8, the maximum and minimum natural frequeny values for the same foundation parameters are obtained for F-F and S-S boundary conditions, respectively. Interestingly, the fundamental mode frequency values of segmented Timoshenko beams on Pasternak foundation having S-S and F-S boundary conditions as well as S-F and F-F boundary conditions are very close. This behaviour is a result of strong foundation parameters of the first segment. As the Winkler spring stiffness of the first segment is high when compared to second and third segments, the displacements close to left end of the first segment are very low irrespective to support condition.

The first three mode shapes of the segmented Timoshenko beams on Pasternak foundation that obtained by using the TMM are presented in Figures 3-6 for S-S, S-F, F-S and F-F boundary conditions, respectively. According to Figures 3-6, the support condition at the left end does not significantly affect the mode shape near the left end due to strong foundation parameter of the first segment. Therefore, especially the fundamental mode shapes of the S-F and F-F beam models look similar. The effects of fixed support condition at the right end of the segmented Timoshenko beams on Pasternak foundation models are more observable when compared to the effects of fixed supports at the left end.

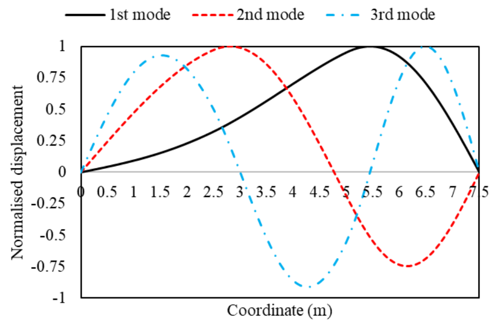

Figure 3. First three mode shapes of segmented Timoshenko beam on Pasternak foundation with S-S boundary conditions (K_W = 50, K_S = 4)

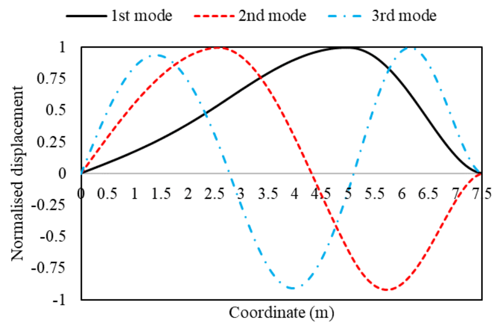

Figure 4. First three mode shapes of segmented Timoshenko beam on Pasternak foundation with S-F boundary conditions (K_W = 50, K_S = 4)

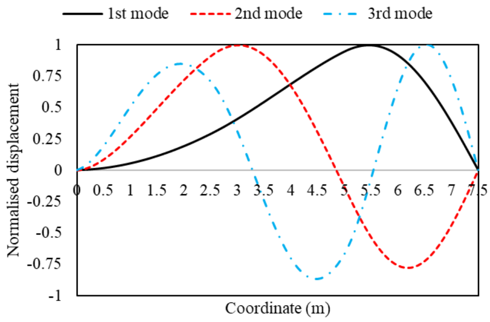

Figure 5. First three mode shapes of segmented Timoshenko beam on Pasternak foundation with F-S boundary conditions (K_W = 50, K_S = 4)

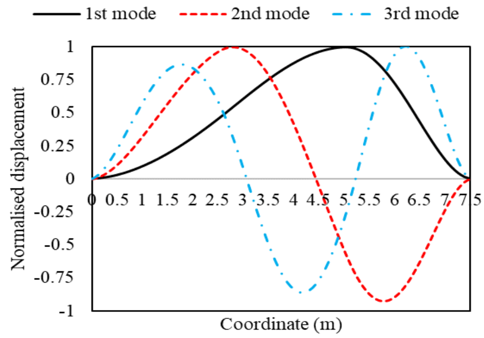

Figure 6. First three mode shapes of segmented Timoshenko beam on Pasternak foundation with F-F boundary conditions (K_W = 50, K_S = 4)

6. Conclusions

The natural frequencies and mode shapes of the segmented Timoshenko beams on Pasternak foundation were obtained by using the transfer matrix formulations. Four different support arrangements were considered. The elastic foundation parameters of the beam segments were taken as differently. The effects of Winkler and Pasternak foundation parameters on the natural frequencies of segmented Timoshenko beams were investigated. The results show that the increment of the natural frequencies by increasing Winkler foundation parameter is more significant in comparison with increase of shear layer parameter of Pasternak foundation.

The effectiveness of TMM on the free vibration analysis of segmented Timoshenko beams on Pasternak foundation was proved. The natural frequency values that obtained using TMM for a simply-supported beam model were validated using the data in literature. For the segmented beam models, the size of global transfer matrix of the whole vibrating system is irrespective to the number of beam segments. The computation time of the root finding algorithm for the calculation of natural frequencies would not be affected by additional elements. Therefore, this study can be extended effectively to more complicated structures such as multi-span multi-segmented Timoshenko beams on Pasternak foundation or considering cracks that can be modeled as rotational springs.

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