

## Solutions for KMM System and Generalized Hyperelastic-Rod Wave Equation

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### Research Article

### ABSTRACT

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In this study, the Kraenkel-Manna-Merle (KMM) system and generalized hyperelastic-rod wave equation have been investigated. For this, generalized Kudryashov method (GKM), which is one of the solution methods of nonlinear evolution equations (NLEEs), has been implemented to KMM system and generalized hyperelastic-rod wave equation. Some solutions have been found for the considered equations and visualized in two and three dimensions using Wolfram Mathematica 12.

## KMM Sistemi ve Genelleştirilmiş Hiperelastik-Rod Dalga Denklemi için Soliton Çözümler

### Araştırma Makalesi

### ÖZ

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Bu çalışmada Kraenkel-Manna-Merle (KMM) sistemi ve genelleştirilmiş hiperelastik-rod dalga denklemi incelenmiştir. Bunun için lineer olmayan evrim denklemlerinin çözüm yöntemlerinden biri olan genelleştirilmiş Kudryashov metodu (GKM), KMM sistemine ve genelleştirilmiş hiperelastik-rod dalga denklemlerine uygulanmıştır. Ele alınan denklemler için bazı çözümler bulunmuş ve Wolfram Mathematica 12 kullanılarak iki ve üç boyutlu olarak görselleştirilmiştir.

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## 1. Introduction

Nonlinear evolution equations (NLEEs) are tackled in quite substantial scientific fields such as physics, biophysics, mathematical physics, optical fibers, mathematical chemistry, hydrodynamics, fluid dynamics, control theory, optics, mechanics, chemical kinematics, biogenetics and so on. With the improving world, NLEEs arise as having more difficult and complex solutions. Solving these equations and finding novel methods form a very important field of study. For this aim, several solution methods such as the  $(G'/G, 1/G)$ -expansion method (Kara and Ünsal, 2022), tanh-coth approach (Ananna et al., 2022), homogeneous balance method (Eslami and Mirzazadeh, 2014),

exponential rational function method (Günay et al., 2021), Darboux-Prelle-Singer methods (Duarte and da Mota, 2021), sine-Gordon expansion method (Tuluçe Demiray and Bayrakci, 2021a) have been presented in the literature by some scientists.

(1+1) dimensional damped- Kraenkel-Manna-Merle (KMM) equation (Kraenkel et al., 2000; Kuetche et al., 2014; Jin and Lin, 2020) is given as:

$$\begin{aligned} w_{xt} - wv_x + sw_x &= 0, \\ v_{xt} + wv_x &= 0. \end{aligned} \tag{1}$$

$w = w(x, t)$  and  $v = v(x, t)$  represent magnetization and the external magnetic fields with respect to the ferrite, whereas  $x$  and  $t$  are the displacement and time variables, respectively, while the parameter  $s$  demonstrates the damping effect (Younas et al., 2022). Eq. (1) defines the nonlinear ultra-short wave pulse motions in saturated ferromagnetic materials with an external field with zero-conductivity (Younas et al., 2022). The damped-KMM system ( $s \neq 0$ ) was proved to be Painlevé property non-integrable. Ignoring the damping effect ( $s = 0$ ), the damped-KMM system is converted into the KMM system (Jin and Lin, 2020).

Ignoring the damping effect ( $s = 0$ ), a lot of solutions have been procured via inverse scattering method (Tchokouansi et al., 2016), generalized G'/G-expansion method (Li and Ma, 2018), bilinear method (Si and Li, 2018), auxiliary equation method (Li and Ma, 2018), Hirota's bilinear method (Nguepjouo et al., 2014), and so on.

Generalized hyperelastic-rod wave equation is presented as (Akcagil et al., 2016; Gözükızıl and Akçağıl, 2013),

$$w_t - w_{xxt} + \alpha w_x + 2\beta w w_x + 3\phi w^2 w_x - \gamma w_x w_{xx} - w w_{xxx} = 0, \tag{2}$$

where  $\alpha, \beta, \phi$  and  $\gamma$  are constants and we assume that  $\phi$  is nonzero. Eq. (2) is utilized to define finite length, small amplitude radial deformation waves in cylindrical compressible hyper-elastic rods. Eq.

(2) also involves substantial physical models. For  $\beta = \frac{3}{2}, \phi = 0$  and  $\gamma = 2$  it can be converted into

Camassa-Holm equation. For  $\beta = \frac{1}{2}, \phi = 0, \alpha = 1$  and  $\gamma = 3$ , it can be converted into Fornberg-

Whitham equation, Besides, for  $\beta = 2, \phi = 0$  and  $\gamma = 3$ , it can be converted into Degasperis-Procesi equation (Akcagil et al., 2016; Gözükızıl and Akçağıl, 2013; Holden and Raynaud, 2007; Bendahmane et al., 2006; Coclite et al., 2005).

Our purpose in this study is to obtain some new solutions of the KMM system and generalized hyperelastic-rod wave equation using the generalized Kudryashov method (GKM) (Barman et al.,

2021; Gurefe, 2020; Tuluçe Demiray and Bayrakci, 2021b; Tuluçe Demiray and Bayrakci, 2021c; Bayrakci et al., 2022; Tuluçe Demiray et al., 2022). The basics of GKM was first presented. Then, the GKM was implemented to the recommended equations and some new soliton solutions were found by using the Wolfram Mathematica 12 package program.

## 2. Material and Method

### 2.1. Structure of GKM

Let's investigate the nonlinear partial differential equation as follow

$$S(w, w_t, w_x, w_{tt}, w_{xt}, w_{xx}, \dots) = 0, \quad (3)$$

where  $w = w(x, t)$  is an unknown function,  $S$  is a polynomial of  $w(x, t)$  and its derivatives with respect to  $x$  and  $t$ .

**Step 1:** The traveling wave solution is assumed as in the following equation,

$$w(x, t) = w(\varepsilon), \quad \varepsilon = k(x - ct), \quad (4)$$

where  $k$  is the wave number and  $c$  is the velocity of the waves .

Using Eq. (4), Eq. (3) is transformed into an ordinary differential equation:

$$T(w, w', w'', w''', \dots) = 0, \quad (5)$$

where  $T$  is a polynomial of  $w$  and its derivatives and superscripts indicate ordinary derivatives according to  $\varepsilon$ .

**Step 2:** Suppose that we imagine the solutions of Eq. (5) as Eq. (6):

$$w(\varepsilon) = \frac{\sum_{k=0}^n a_k Z^k(\varepsilon)}{\sum_{l=0}^m b_l Z^l(\varepsilon)} = \frac{G[Z(\varepsilon)]}{H[Z(\varepsilon)]}, \quad (6)$$

where  $Z$  is  $\frac{1}{1 \pm e^\varepsilon}$ . We must specify that  $Z$  is the solution to Eq. (7),

$$Z' = Z^2 - Z. \quad (7)$$

Using Eq. (6), the following derivatives are obtained,

$$w'(\varepsilon) = \frac{G'Z'H - GH'Z'}{H^2} = Z' \left[ \frac{G'H - GH'}{H^2} \right] = (Z^2 - Z) \left[ \frac{G'H - GH'}{H^2} \right], \quad (8)$$

$$w''(\varepsilon) = \frac{Z^2 - Z}{H^2} \left[ \begin{aligned} &(2Z - 1)(G'H - GH') \\ &+ \frac{Z^2 - Z}{H} \left[ H(G''H - GH'') - 2H'G'H + 2G(H')^2 \right] \end{aligned} \right], \quad (9)$$

where  $G' = \frac{dG}{dZ}$ ,  $H' = \frac{dH}{dZ}$ ,  $G'' = \frac{d^2G}{dZ^2}$ ,  $H'' = \frac{d^2H}{dZ^2}$ .

**Step 3:** The solution of the nonlinear ordinary differential equation given by Eq. (5) is sought according to the GKM as follows:

$$w(\varepsilon) = \frac{a_0 + a_1Z + a_2Z^2 + \dots + a_nZ^n}{b_0 + b_1Z + b_2Z^2 + \dots + b_mZ^m}. \quad (10)$$

The homogeneous balance principle is used to find the values of  $m$  and  $n$  in Eq. (6). For this purpose, a balance is established between the highest order derivative and the highest order nonlinear term in Eq. (5).

**Step 4:** We put Eq. (6) into Eq. (5). Thus we get a polynomial  $R(Z)$  of  $Z$ . Then equalizing the overall coefficients of  $R(Z)$  to zero, we obtain an algebraic equation system. By solving the resulting system, we designate  $k, c$  and the variable coefficients of  $a_0, a_1, a_2, \dots, a_n, b_0, b_1, b_2, \dots, b_m$ . Finally, we can have the exact solutions of Eq. (5).

### 3. Results

#### 3.1. Practice of GKM to the KMM Equation

To get some solutions of Eq. (1), we take into account the following equality:

$$w(x, t) = w(\varepsilon), v(x, t) = v(\varepsilon), \varepsilon = k(x - ct), \quad (k \neq 0, c \neq 0). \quad (11)$$

Putting Eq. (11) into Eq. (1), we find the following equalities

$$-ck^2w''(\varepsilon) - kwv'(\varepsilon) = 0, \quad (12)$$

$$kw(\varepsilon)w'(\varepsilon) - ck^2v''(\varepsilon) = 0. \quad (13)$$

Hence

$$v''(\varepsilon) = \frac{w(\varepsilon)w'(\varepsilon)}{ck}, \quad (14)$$

is found. By integrating Eq. (14) with respect to  $\varepsilon$ ,

$$v'(\varepsilon) = \frac{w^2(\varepsilon)}{2ck} + c_0, \quad (15)$$

is obtained. Here  $c_0$  is the integration constant. If Eq. (15) is inserted into Eq. (12), ordinary differential equation can be expressed in the following form

$$2cc_0kw(\varepsilon) + w^3(\varepsilon) + 2c^2k^2w''(\varepsilon) = 0. \quad (16)$$

If the integration constant of Eq. (15) is taken as zero and by integrating Eq. (15) with respect to  $\varepsilon$ , we get

$$v(\varepsilon) = \frac{w^3(\varepsilon)}{6ck} + c_0w(\varepsilon). \quad (17)$$

If the balance principle is applied between  $w^3(\varepsilon)$  and  $w''(\varepsilon)$  in Eq. (16), we get

$$n - m + 2 = 3n - 3m \Rightarrow n = m + 1. \quad (18)$$

If we choose  $m = 1$  and  $n = 2$ , we get

$$w(\varepsilon) = \frac{a_0 + a_1Z + a_2Z^2}{b_0 + b_1Z}, \quad (19)$$

$$w'(\varepsilon) = (Z^2 - Z) \left[ \frac{(a_1 + 2a_2Z)(b_0 + b_1Z) - b_1(a_0 + a_1Z + a_2Z^2)}{(b_0 + b_1Z)^2} \right], \quad (20)$$

$$w''(\varepsilon) = \frac{Z^2 - Z}{(b_0 + b_1Z)^2} (2Z - 1) \left[ (a_1 + 2a_2Z)(b_0 + b_1Z) - b_1(a_0 + a_1Z + a_2Z^2) \right] \\ + \frac{(Z^2 - Z)^2}{(b_0 + b_1Z)^3} \left[ 2a_2(b_0 + b_1Z)^2 - 2b_1(a_1 + 2a_2Z)(b_0 + b_1Z) + 2b_1^2(a_0 + a_1Z + a_2Z^2) \right]. \quad (21)$$

**Case 1:**

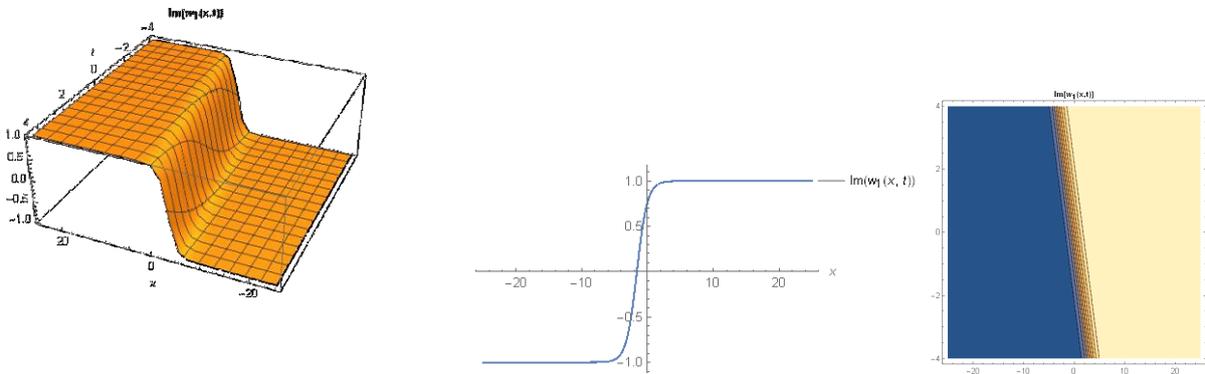
$$a_0 = -2ib_0c_0, a_1 = -\frac{a_2}{2} + 4ib_0c_0, a_2 = a_2, b_0 = b_0, b_1 = -\frac{ia_2}{4c_0}, k = \frac{2c_0}{c}. \quad (22)$$

By placing Eq. (22) in Eq. (19), we get the dark soliton solutions of Eq. (1)

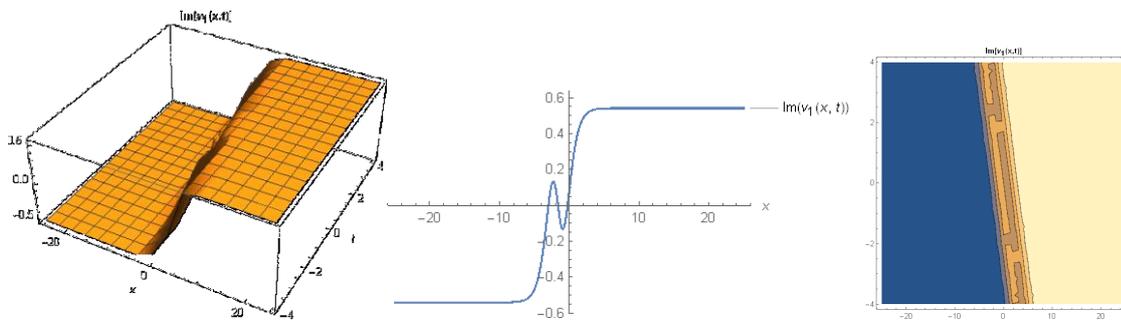
$$w_1(x,t) = -2ic_0 \tanh\left[\frac{(x-ct)c_0}{c}\right], \quad (23)$$

$$v_1(x,t) = -2ic_0^2 \tanh\left[\frac{(x-ct)c_0}{c}\right] + \frac{4ic_0^3}{3ck} \tanh^3\left[\frac{(x-ct)c_0}{c}\right]. \quad (24)$$

2D and 3D graphs of the solutions (23) and (24) are demonstrated with contour simulations in Figure 1 and Figure 2.



**Figure 1.** 3D, contour plots of solution (23) for  $c = -0.8, c_0 = -0.5, -25 \leq x \leq 25$ , values with  $-4 \leq t \leq 4$  range and 2D plot of solution for  $t = 2$  with these values.



**Figure 2.** 3D, contour plots of solution (24) for  $c = -0.8, c_0 = -0.5, k = 0.2, -25 \leq x \leq 25$  values with  $-4 \leq t \leq 4$  range and 2D plot of solution for  $t = 2$  with these values.

**Case 2:**

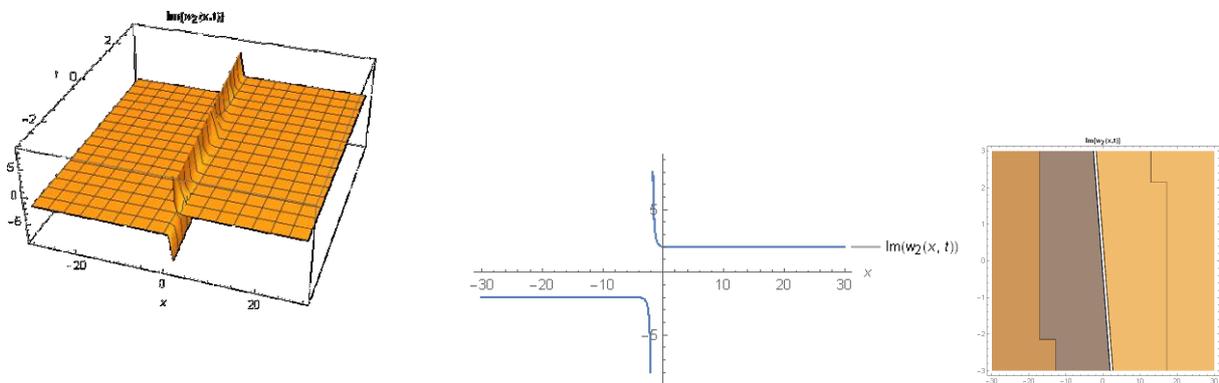
$$a_0 = \frac{1}{2}ib_1c_0, \quad a_1 = -ib_1c_0, \quad a_2 = ib_1c_0, \quad b_0 = -\frac{b_1}{2}, \quad b_1 = b_1, \quad k = \frac{c_0}{2c}. \quad (25)$$

By placing Eq. (25) in Eq. (19), we get the dark soliton solutions of Eq. (1)

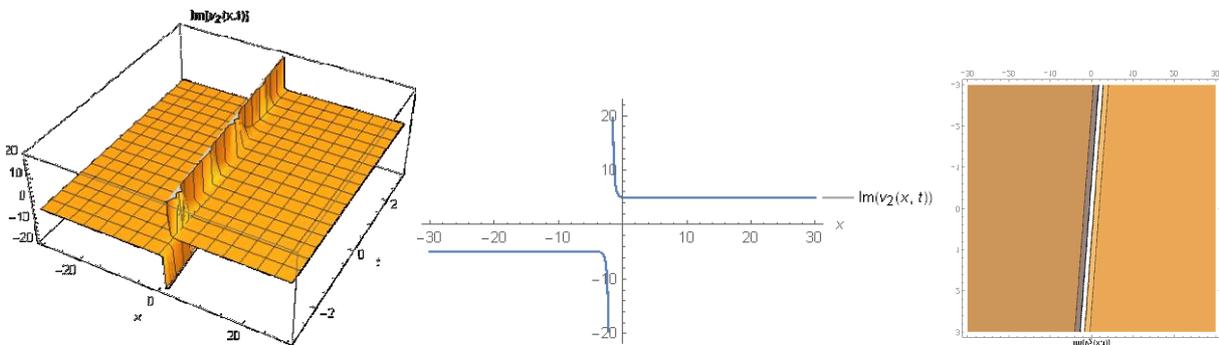
$$w_2(x,t) = -ic_0 \coth \left[ \frac{(x-ct)c_0}{2c} \right], \quad (26)$$

$$v_2(x,t) = -ic_0^2 \coth \left[ \frac{(x-ct)c_0}{2c} \right] + \frac{ic_0^3}{6ck} \coth^3 \left[ \frac{(x-ct)c_0}{2c} \right]. \quad (27)$$

2D and 3D graphs of the solutions (26) and (27) are demonstrated with contour simulations in Figure 3 and Figure 4.



**Figure 3.** 3D, contour plots of solution (26) for  $c = -0.75, c_0 = 2, -30 \leq x \leq 30$ , values with  $-3 \leq t \leq 3$  range and 2D plot of solution for  $t = 2.5$  with these values.



**Figure 4.** 3D, contour plots of solution (27) for  $c = -0.75, c_0 = 2, k = 2, -30 \leq x \leq 30$ , values with  $-3 \leq t \leq 3$  range and 2D plot of solution for  $t = 2.5$  with these values.

**Case 3:**

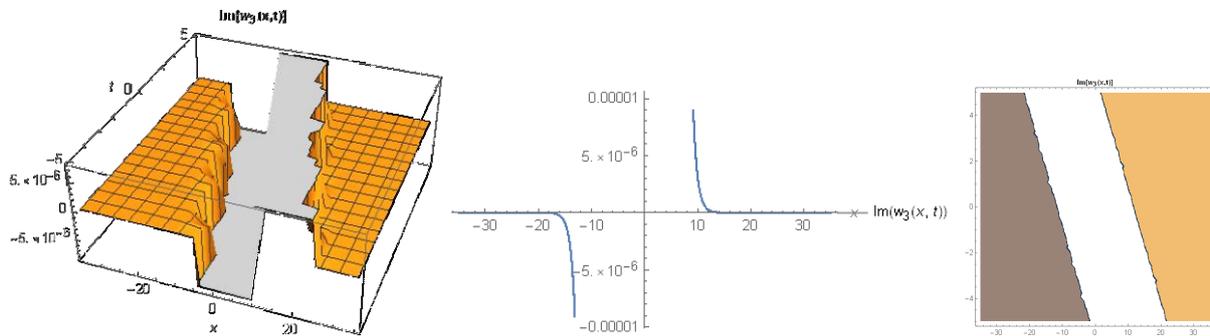
$$a_0 = 0, \quad a_1 = -a_2, \quad a_2 = a_2, \quad b_0 = \frac{ia_2}{4c_0}, \quad b_1 = -\frac{ia_2}{2c_0}, \quad k = -\frac{c_0}{c}. \quad (28)$$

By placing Eq. (28) in Eq. (19), we get the bright soliton solutions of Eq. (1)

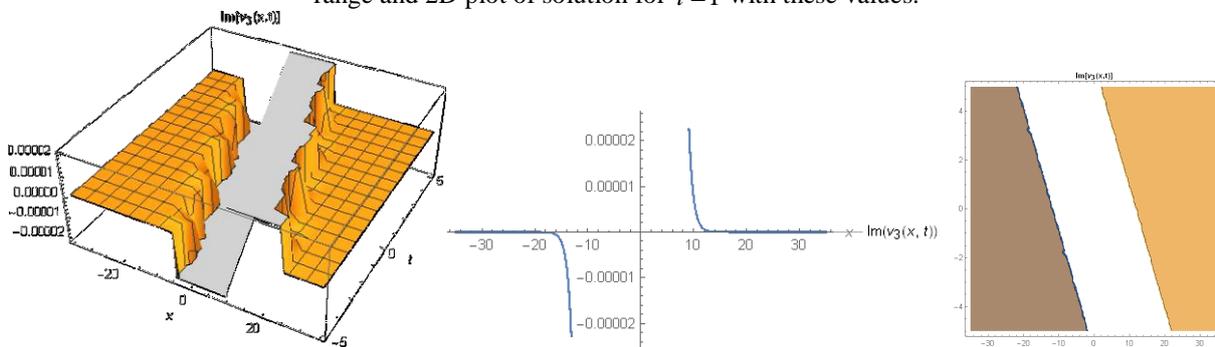
$$w_3(x, t) = -2ic_0 \operatorname{csch} \left[ \frac{(x-ct)c_0}{c} \right], \quad (29)$$

$$v_3(x, t) = -2ic_0^2 \operatorname{csch} \left[ \frac{(x-ct)c_0}{c} \right] + \frac{4ic_0^3}{3ck} \operatorname{csch}^3 \left[ \frac{(x-ct)c_0}{c} \right]. \quad (30)$$

2D and 3D graphs of the solutions (29) and (30) are demonstrated with contour simulations in Figure 5 and Figure 6.



**Figure 5.** 3D, contour plots of solution (29) for  $c = -2, c_0 = 2.5, -35 \leq x \leq 35$ , values with  $-5 \leq t \leq 5$  range and 2D plot of solution for  $t = 1$  with these values.



**Figure 6.** 3D, contour plots of solution (30) for  $c = -2, c_0 = 2.5, k = 0.4, -35 \leq x \leq 35$ , values with  $-5 \leq t \leq 5$  range and 2D plot of solution for  $t = 1$  with these values.

### 3.2. Practice of GKM to the Hyperelastic-Rod Wave Equation

To get some soliton solutions of Eq. (2), we take into account the following equality:

$$w(x, t) = w(\varepsilon), \quad \varepsilon = x - ct. \quad (31)$$

Putting Eq. (31) into Eq. (2), we find the following

$$-cw' + cw''' + \alpha w' + 2\beta ww' + 3\phi w^2 w' - \gamma w' w'' - ww''' = 0. \quad (32)$$

By integrating Eq. (32) with respect to  $\varepsilon$  and taking the integration constant zero, we get

$$(-c + \alpha)w + cw'' + \beta w^2 + \phi w^3 - \frac{\gamma - 1}{2}(w')^2 - ww'' = 0. \quad (33)$$

If the balance principle is applied between  $w^3$  and  $ww''$  in Eq. (33), we get

$$n - m + n - m + 2 = 3n - 3m \Rightarrow n = m + 2. \quad (34)$$

If we choose  $m = 1$  and  $n = 3$ , we get

$$w(\varepsilon) = \frac{a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3}{b_0 + b_1 Z}, \quad (35)$$

$$w'(\varepsilon) = (Z^2 - Z) \left[ \frac{(a_1 + 2a_2 Z + 3a_3 Z^2)(b_0 + b_1 Z) - b_1(a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3)}{(b_0 + b_1 Z)^2} \right], \quad (36)$$

$$w''(\varepsilon) = \frac{Z^2 - Z}{(b_0 + b_1 Z)^2} (2Z - 1) \left[ (a_1 + 2a_2 Z + 3a_3 Z^2)(b_0 + b_1 Z) - b_1(a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3) \right] \\ + \frac{(Z^2 - Z)^2}{(b_0 + b_1 Z)^3} \left[ (b_0 + b_1 Z)^2 (2a_2 + 6a_3 Z) - 2b_1(b_0 + b_1 Z)(a_1 + 2a_2 Z + 3a_3 Z^2) + 2b_1^2(a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3) \right]. \quad (37)$$

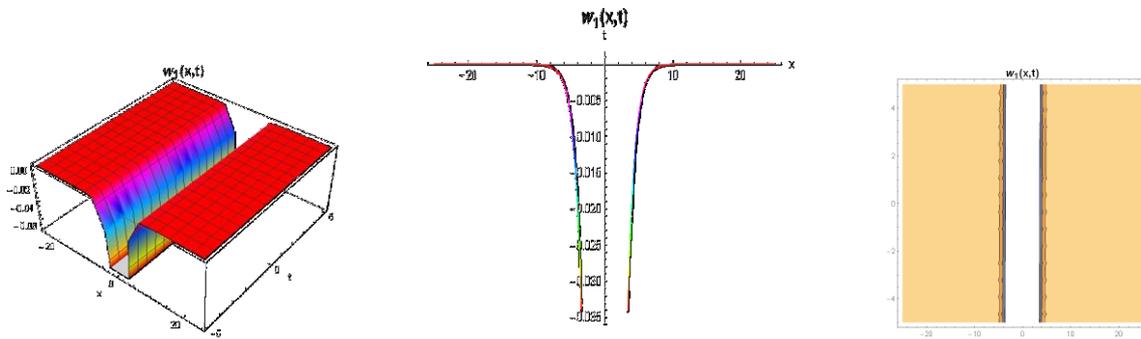
**Case 1:**

$$\begin{aligned}
a_0 = 0, a_1 = -\frac{2(2+\gamma)b_0}{\phi}, a_2 = \frac{2(2+\gamma)(b_0-b_1)}{\phi}, b_0 = b_0, b_1 = b_1, \\
a_3 = \frac{2(2+\gamma)b_1}{\phi}, \alpha = 0, c = -\frac{(-1+2\beta-\gamma)(2+\gamma)}{6\phi}.
\end{aligned}
\tag{38}$$

By placing Eq. (38) in Eq. (35), we get the hyperbolic solution of Eq. (2)

$$w_1(x,t) = -\frac{2+\gamma}{\phi + \phi \cosh \left[ x + \frac{t(-1+2\beta-\gamma)(2+\gamma)}{6\phi} \right]}
\tag{39}$$

2D and 3D graphs of the solution (39) is demonstrated with contour simulations in Figure 7.



**Figure 7.** 3D, contour plots of solution (39) for  $\gamma = 0.2, \phi = 4, \beta = 0.5, -25 \leq x \leq 25$ , values with  $-5 \leq t \leq 5$  range and 2D plot of solution for  $t = 2$  with these values.

## Case 2:

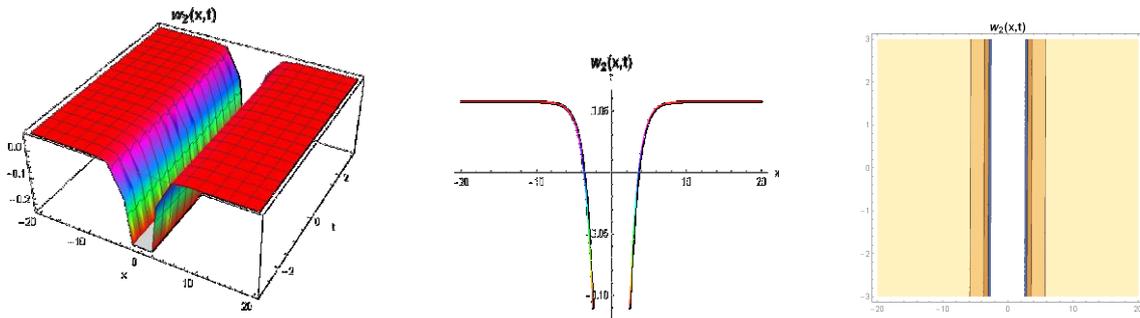
$$\begin{aligned}
a_0 = \frac{b_0(3(2-\beta+\gamma)\phi b_1 - \sqrt{3}\Lambda)}{12\phi^2 b_1}, \quad \Lambda = \sqrt{(4+3\beta^2 - \gamma^2 + 2\beta(2+\gamma))\phi^2 b_1^2}, \\
a_1 = -\frac{24(2+\gamma)\phi b_0 + 3(-2+\beta-\gamma)\phi b_1 + \sqrt{3}\sqrt{(4+\beta(4+3\beta) + 2\beta\gamma - \gamma^2)\phi^2 b_1^2}}{12\phi^2}, \\
a_2 = \frac{2(2+\gamma)(b_0-b_1)}{\phi}, a_3 = \frac{2(2+\gamma)b_1}{\phi}, b_0 = b_0, b_1 = b_1, \\
\alpha = \frac{3(\beta^2 - (2+\gamma)^2 - 2\beta(3+\gamma))\phi b_1 + \sqrt{3}(4+\beta+3\gamma)\Lambda}{24\phi^2 b_1}, \\
c = \frac{-((1+\gamma)(2+\gamma) + \beta(5+\gamma))\phi b_1 + \sqrt{3}(1+\gamma)\Lambda}{12\phi^2 b_1}.
\end{aligned}
\tag{40}$$

By placing Eq. (40) in Eq. (35), we get the hyperbolic solution of Eq. (2)

$$w_2(x,t) = -\frac{1}{24\phi^2} \left( \frac{6\phi(6+\beta+3\gamma+(-2+\beta-\gamma)\cosh[x-ct])}{1+\cosh[x-ct]} + \chi \right), \quad (41)$$

where  $\chi = \frac{2\sqrt{3}\Lambda}{b_1}$ ,  $c = \frac{-((1+\gamma)(2+\gamma)-\beta(5+\gamma))\phi b_1 + \sqrt{3}(1+\gamma)\Lambda}{12\phi^2 b_1}$ .

2D and 3D graphs of the solution (41) is demonstrated in with contour simulations in Figure 8.



**Figure 8.** 3D, contour plots of solution (41) for  $\gamma = 0.3, \phi = 2, \beta = 0.4, b_1 = 2$ ,  $-20 \leq x \leq 20$ , values with  $-3 \leq t \leq 3$  range and 2D plot of solution for  $t = 1$  with these values.

**Case 3:**

$$a_0 = 0, a_1 = a_1, a_2 = a_2, a_3 = -a_2, b_0 = 0, b_1 = -\frac{\phi a_2}{2(2+\gamma)},$$

$$\alpha = \frac{2(2+\gamma)a_1(12(2+\gamma)a_1^2 + 6(1+\gamma)a_1a_2 + \gamma a_2^2)}{\phi a_2^2(6a_1 + a_2)}, \quad (42)$$

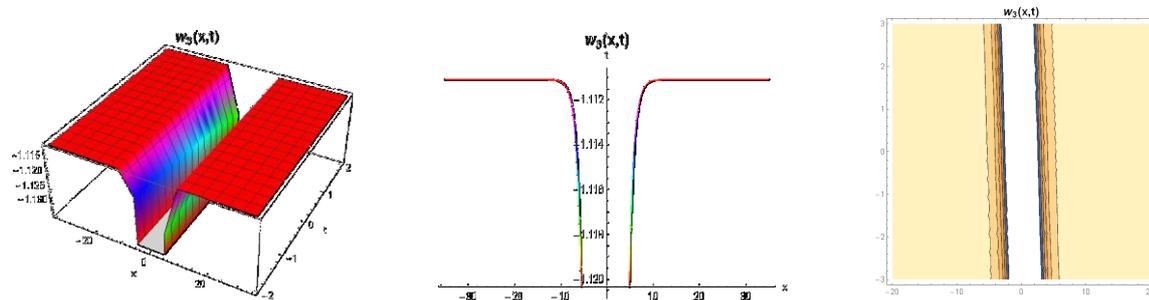
$$\beta = \frac{1}{2} \left( 1 + \gamma + \frac{8(2+\gamma)a_1}{a_2} - \frac{2(-1+\gamma)a_1}{6a_1 + a_2} \right), c = \frac{(-2+\gamma+\gamma^2)a_1(4a_1 + a_2)}{\phi a_2(6a_1 + a_2)}.$$

By placing Eq. (42) in Eq. (35), we get the bright soliton solution of Eq. (2)

$$w_3(x,t) = \Omega \left[ 4a_1 + a_2 \operatorname{sech}^2 \left[ \frac{1}{2} \left( x - \frac{t(-2+\gamma+\gamma^2)a_1(4a_1 + a_2)}{\phi a_2(6a_1 + a_2)} \right) \right] \right], \quad (43)$$

where  $\Omega = -\frac{(2+\gamma)}{2\phi a_2}$ .

2D and 3D graphs of the solution (43) is demonstrated with contour simulations in Figure 9.



**Figure 9.** 3D, contour plots of solution (43) for  $\gamma = 0.5, \phi = 3, a_1 = 2, a_2 = 3, -35 \leq x \leq 35$ , values with  $-2 \leq t \leq 2$  range and 2D plot of solution for  $t = 1.5$  with these values.

#### 4. Conclusion

In this study, GKM was applied to acquire solutions of Kraenkel-Manna-Merle (KMM) system and generalized hyperelastic-rod wave equation. Thus, solutions of KMM system and generalized hyperelastic-rod wave equation were procured such as dark soliton, bright soliton and hyperbolic solutions. In addition, for some certain values, 3D and 2D graphical representations of these solutions were given with contour simulations with the help of Wolfram Mathematica 12. As far as we know, GKM has not been applied to KMM system and generalized hyperelastic-rod wave equation before. In the light of the results, we deduce that GKM is an effective method in NLEEs calculation.

#### Statement of Conflict of Interest

Authors have declared no conflict of interest.

#### Author’s Contributions

The contribution of the authors is equal.

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